Optimal Stopping in a Dynamic Salience Model

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Motivation

Many important choices are dynamic:

- when to enter the job market or to retire,
- when to stop searching for a job, a house, or a spouse,
- when to sell an asset.

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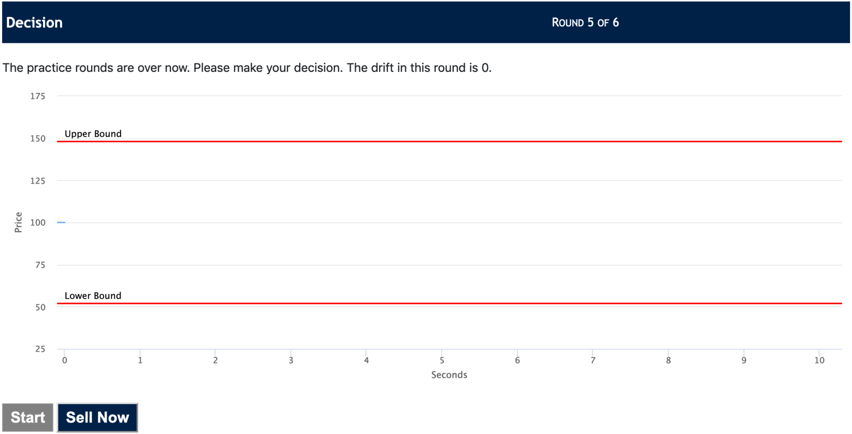
Welfare implications: neglect studying, holding an asset for too long, . . .

No commitment and endogenous skewness can result in excessive gambling



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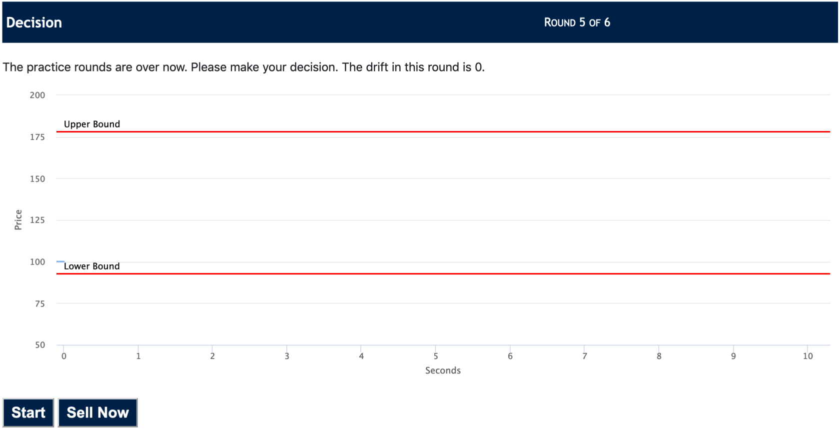

→ The strategy yields a right-skewed distribution (or "loss-exit" strategy).

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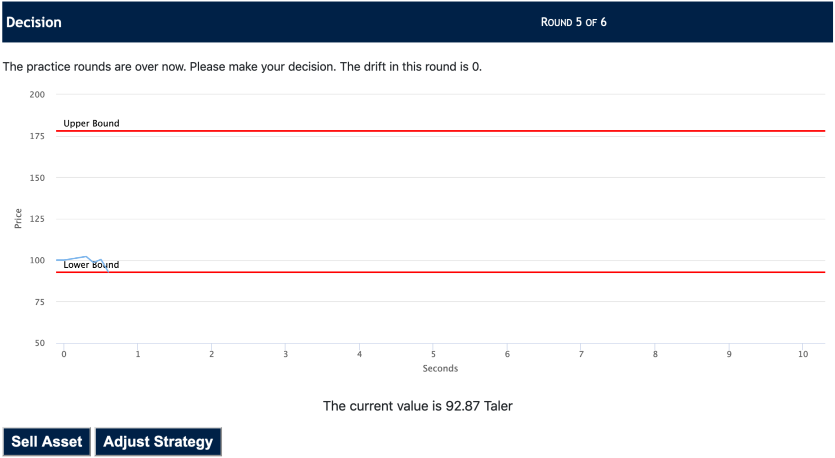
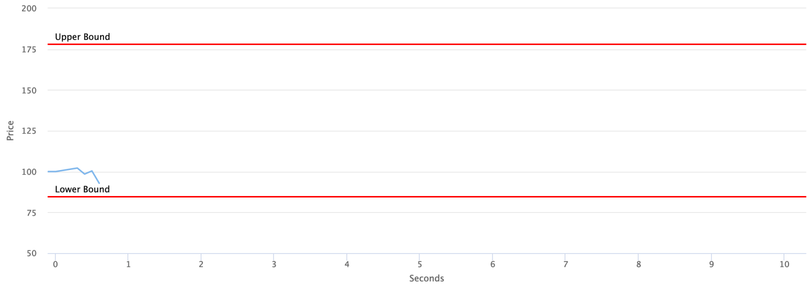


$\longrightarrow$ Does the agent actually exit? Or does she come up with a new strategy?

No commitment and endogenous skewness can result in excessive gambling

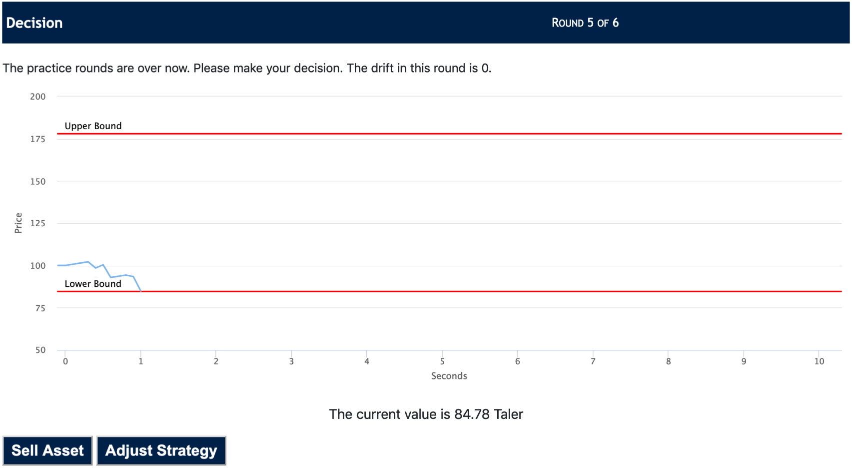
Decision ROUND 5 OF 6

The practice rounds are over now. Please make your decision. The drift in this round is 0.



Start

No commitment and endogenous skewness can result in excessive gambling



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   - Conditional on gambling, most people choose mostly loss-exit strategies.
   - People reveal consistent skewness preferences in static and dynamic choices.

Summary: ST makes precise predictions and describes actual behavior quite well.

Related literature

Literature on the modeling of/testing for skewness preferences:

Theory: Kahneman and Tversky (1979, 1992), Menezes et al. (1980), Bordalo et al. (2012)

Experiments: Ebert (2015), Dertwinkel-Kalt and Köster (2020)

$\longrightarrow$ We propose a dynamic version of salience theory of choice under risk.

$\longrightarrow$ Skewness preferences revealed in static and dynamic problems are consistent.

Theoretical and experimental literature on behavioral stopping:

Theory: Machina (1989), Karni and Safra (1990), Barberis (2012), Xu and Zhou (2013), Ebert and Strack (2015, 2018), Duraj (2019)

Experiments: Imas (2016), Imas et al. (2017), Fischbacher et al. (2017), Strack and Viefers (2019), Heimer et al. (2020)

$\longrightarrow$ Derive and test the non-parametric salience predictions on stopping behavior.

The Model

Salience theory of choice under risk (Bordalo et al., 2012)

Choice between random variables X and Y with joint CDF F. Choose X iff

$$\int (x-y)\cdot\sigma(x,y)\;dF(x,y) > 0,$$

where the salience function σ is symmetric, bounded, and satisfies:

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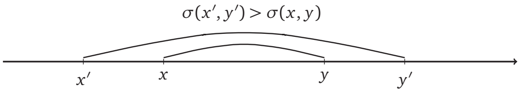
Contrast effect: *differences attract a decision maker's attention.*



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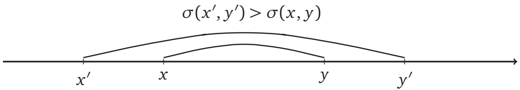
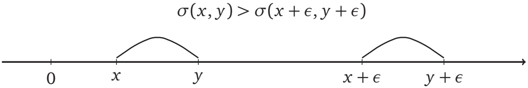


Level effect: *a given contrast is less salient at a higher outcome level.*



Stochastic process, stopping strategies, and solution concept

As in Ebert and Strack (2015), the asset's price evolves according to an ABM

$$dX_t = \mu dt + \nu dW_t \quad \text{with} \quad X_0 = x \quad \text{and} \quad X_t \geq 0, \tag{1}$$

where $(W_t)_{t \geq 0}$ is a BM, $\mu \in \mathbb{R}$ gives the drift, and $\nu \in \mathbb{R}_+$ the volatility. The process is non-negative and absorbing in zero, and has a finite expiration date $T < \infty$.

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Subjects can choose two-threshold stopping strategies

Solution concept: a naive decision rule à la Ebert and Strack (2015)
"At every point in time the naive [salient thinker] looks for some strategy that brings her higher [salience-weighted utility] than stopping immediately. If such a strategy exists, [he] holds on to the investment — irrespective of [his] earlier plan."

Theoretical Results

An illustrative example

Denote the current wealth level y, and consider gambling with thresholds a and b.

Zero drift: salient thinker gambles if and only if $\sigma(a, y) < \sigma(b, y)$.

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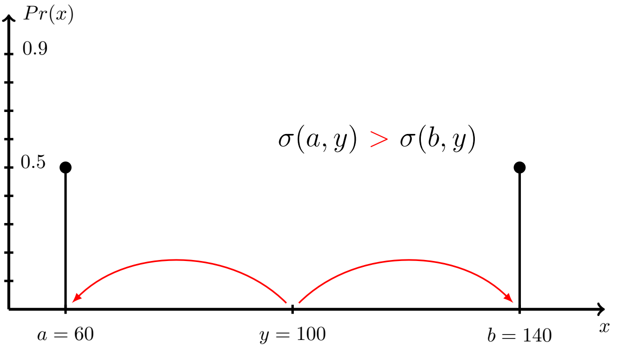

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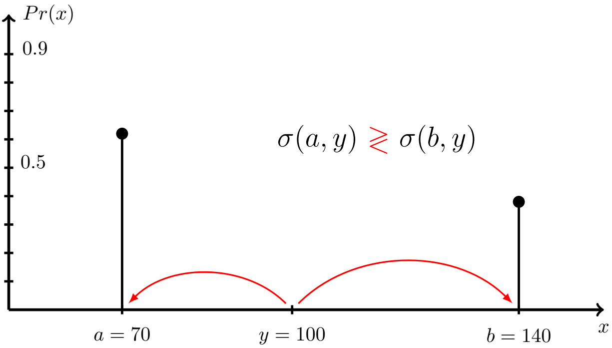

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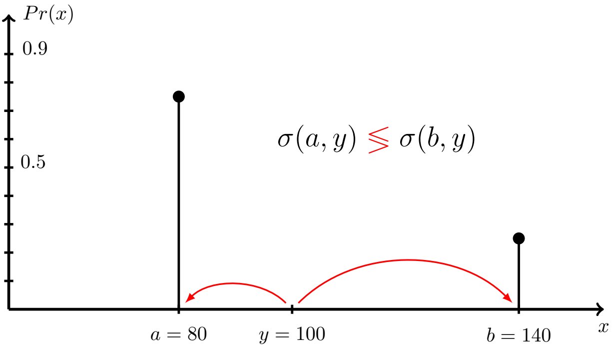

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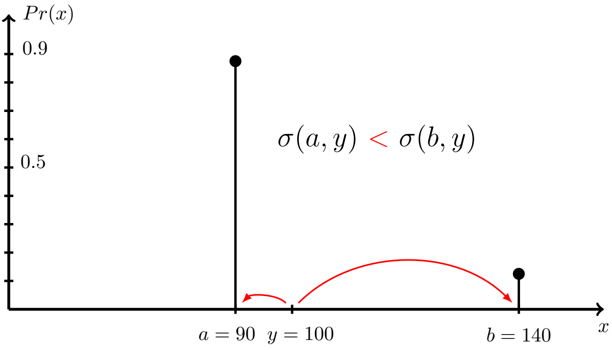

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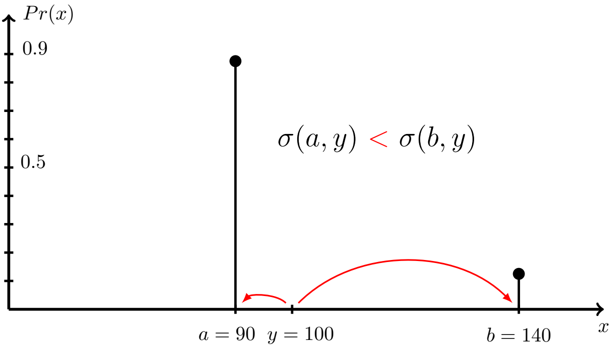


Proposition 1

A salient thinker plays a process with zero drift.

Overview of the salience predictions on stopping behavior

Prediction 1

The share of "immediate sellers" monotonically decreases in the drift.

→ Distinguishes ST from EUT (w/ concave utility), which predicts no gambling, and from CPT, which yields *never stopping* irrespective of the drift.

Prediction 2

Conditional on not selling the asset, subjects choose a loss-exit strategy.

Prediction 3

Consistent skewness preferences across static and dynamic choices. Details

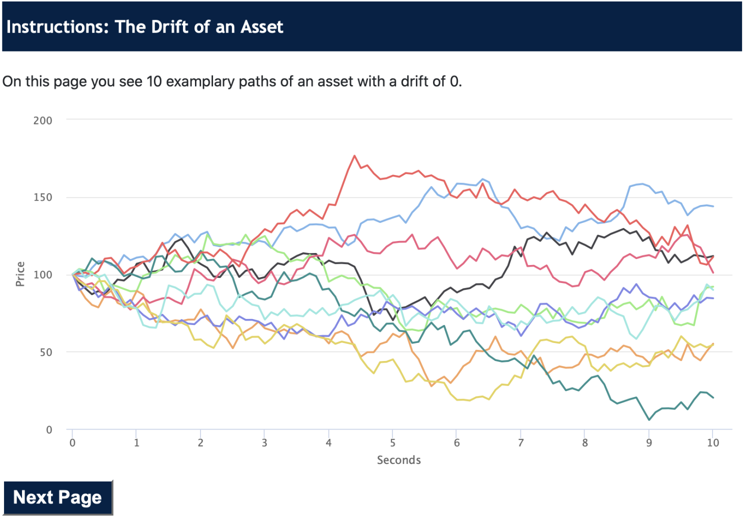
Experimental Design

Implementation

- $n = 158$ in 5 sessions in Cologne in Jan 2020.
- 10 ECU $=$ 1 Euro.
- $T = 10$ sec and subjects could always pause the process.
- 6 processes with 0, -1, -3, -5, -10, -20 as drifts per sec.
- Order of drifts randomized at the subject level.
- Verbal explanation of the stochastic process. Details
- Subsequent test for static skewness preferences (12 questions).

Pre-registered at AEA Registry: `https://doi.org/10.1257/rct.5359-1.0`.

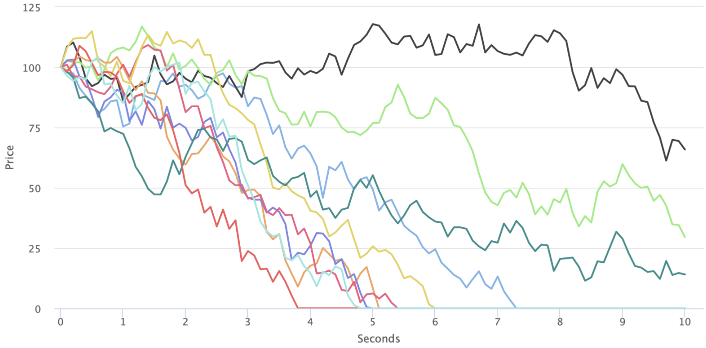
Before making a selling decision, subjects could sample from the underlying process



Before making a selling decision, subjects could sample from the underlying process

Instructions: The Drift of an Asset

On this page you see 10 examplary paths of an asset with a drift of -10.



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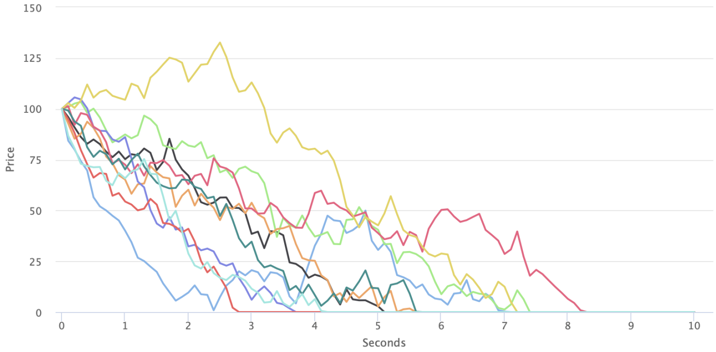

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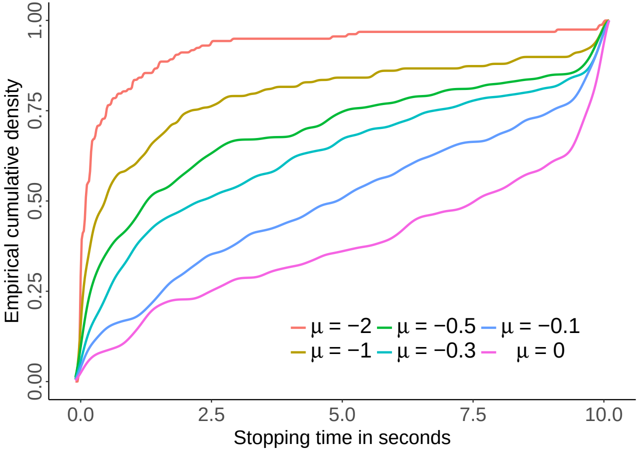
Main Experimental Results

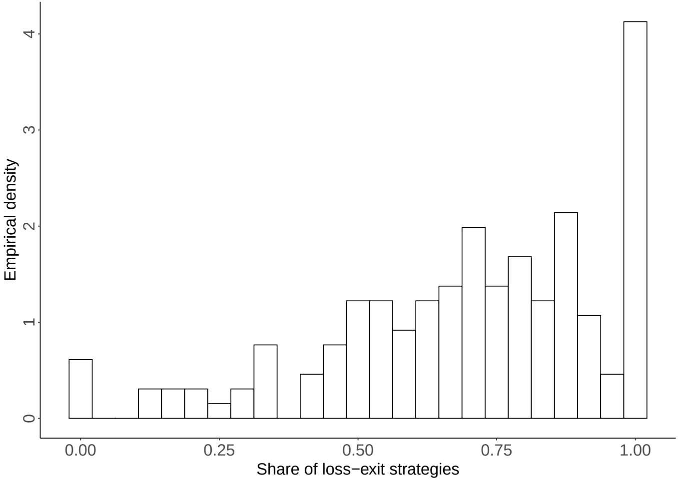
Result 1

Subjects stop earlier for processes with a more negative drift. In particular, the share of subjects selling immediately monotonically decreases in the drift. Details



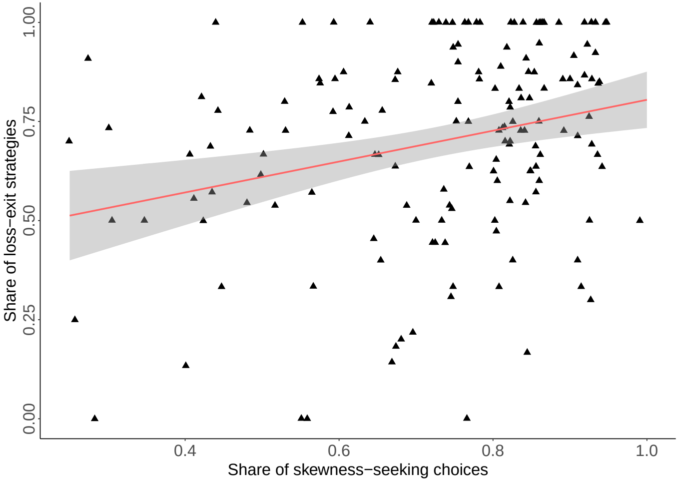
Result 2

Conditional on not selling the asset immediately, a majority of subjects initially chooses a loss-exit strategy. The median subject chooses 73% loss-exit strategies.



Result 3

Consistent static and dynamic skewness preferences ($\rho = 0.39$, p-value < 0.001).



Additional Results

Adjustments of the initial strategies

- Only 1% of the subjects did never adjust the initial strategy.
- The median subject adjusts her strategy once per round, on average.
- Across all drifts, around 85% of all processes are stopped later than planned.
- Subjects mostly switch from a loss-exit to another loss-exit strategy:

		To: Loss-Exit	To: Gain-Exit
From	**Loss-Exit**	63.31%	10.31%
From	**Gain-Exit**	12.01%	14.37%

- We also find disposition-effect-like behavior consistent with ST. Details

Discussion

Key take-aways

1. Endogenous skewness matters, but stopping behavior is sensitive to the drift.
2. Further applications: job search, striving for an elusive goal, ...
3. People reveal consistent skewness preferences in static and dynamic choices.
4. ST is a promising candidate for unified theory of static and dynamic choice.

In discrete, finite time CPT predicts stopping close to the expiration date

Let the process be given by a fair coin that is tossed repeatedly T times.

Whenever the coin comes up heads (tails) the value goes up (down) by 10 cents.

Consider Tversky and Kahneman's (1992) representative CPT agent:

$$v(x) = \begin{cases} (x-r)^\alpha & \text{if } x \geq r, \\ -\lambda(-(x-r))^\alpha & \text{if } x < r, \end{cases} \qquad \text{and} \qquad w(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}}$$

with $\alpha = 0.88$, $\lambda = 2.25$, and $\delta = 0.65$. When does this agent stop?

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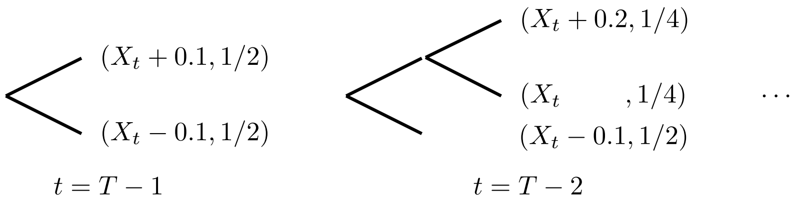
Consider the stopping time $\tau_{a,b}$ with $a = X_t - 0.1$ and $b = X_t + (T-t)0.1$:



In discrete, finite time CPT predicts stopping close to the expiration date – cont'd

Result: the representative CPT agent might stop eventually, but away from the reference point already a few coin tosses suffice for her to gamble. Precisely:

1 coin toss remaining: stop if and only if $X_{T-1} \in \{r, r+0.1, \ldots, r+(T-1)0.1\}$.

2 coin tosses remaining: stop if and only if $X_{T-2} \in \{r, r+0.1, r+0.2, r+0.3\}$.

3 coin tosses remaining: stop if and only if $X_{T-3} \in \{r, r+0.1\}$.

4 coin tosses remaining: stop if and only if $X_{T-4} = r$.

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Verbal explanation of the stochastic process

"In this experiment you will see assets of varying profitability. How profitable an asset is in the long run is described by the drift of the asset. The drift denotes the average change in the value of the process per second."

and

"A positive drift implies that the asset will increase in value in the long run, while a negative drift implies that the asset will decrease in value in the long run. Notice that the value of the asset varies. Hence, even an asset with a negative drift sometimes increases in value."

and

"Independent of the drift, the value of the asset can, in principle, become arbitrarily large. The probability that the asset's value indeed becomes very large is the smaller the more negative the drift is. But even an asset with a very negative drift can attain a very large value."

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Relation between skewness preferences revealed in static and dynamic environments

Lottery	Exp. Value	Skewness
(37.5, 80%; 0, 20%)	30	-1.5
(41.25, 64%; 10, 36%)	30	-0.6
(45, 50%; 15, 50%)	30	0
(60, 20%; 22.5, 80%)	30	1.5
(75, 10%; 25, 90%)	30	2.7
(135, 2%; 27.85, 98%)	30	6.9

Table: *Test for static skewness preferences (from Dertwinkel-Kalt and Köster, 2020).*

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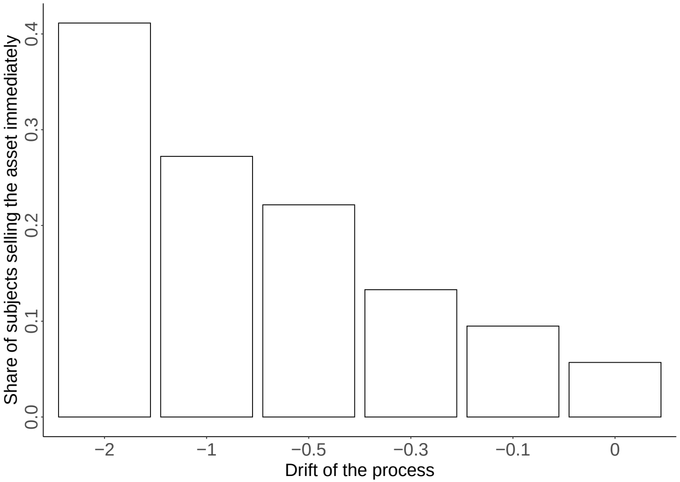

Share of subjects selling the asset immediately
0.0
0.1
0.2
0.3
0.4
−2
−1
−0.5
−0.3
−0.1
0
Drift of the process

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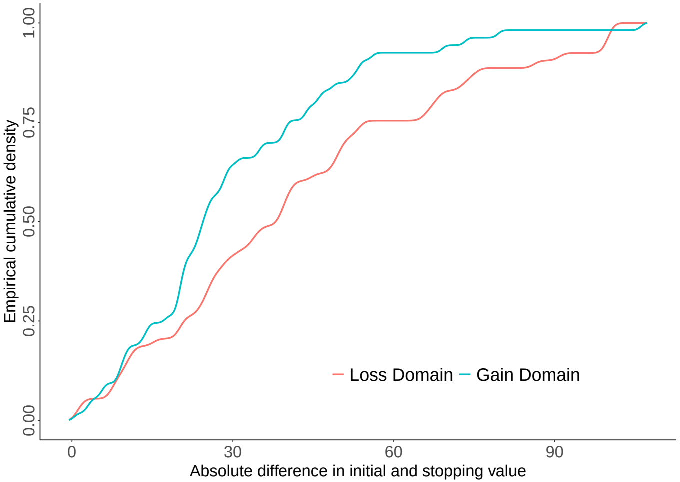

Empirical cumulative density
1.00
0.75
0.50
0.25
0.00
0
30
60
90
Absolute difference in initial and stopping value
Loss Domain
Gain Domain

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